

# Scatter and Glow - BCH Formulas

Pensieve Header: Several BCH mod CC formulas.

## ■ Failed Tests

```
In[1]:= Dynamic[$FailLog]
```

```
Out[1]= {On {2009, 8, 4, 10, 59, 17.0110000} failed in 0 == 1}
```

## ■ Testing Code

```
In[2]:= SetAttributes[{Test, Expect}, {HoldAll}];
Test[expr_] := If[TrueQ[Check[expr, False]], True,
  If[Head[$FailLog] != List, $FailLog = {}];
  AppendTo[$FailLog,
    "On " <> ToString[Date[]] <> " failed in " <> ToString[HoldForm[expr]]];
  Print[Last[$FailLog]]
];
Expect[val_, expr_] := If[TrueQ[Test[val == expr]], val];

SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\ScatterAndGlow"]
<< ScatterAndGlow.m
```

```
Out[5]= C:\drorbn\AcademicPensieve\Projects\ScatterAndGlow
```

## ■ Test Test

```
In[7]:= Test[0 == 1]
```

```
On {2009, 8, 4, 10, 59, 17.0110000} failed in 0 == 1
```

## ■ $\exp(x)\exp(y)=\exp(x+y+bch1)$

```
In[8]:= (lhs1 = S[sigma[1, 3], sigma[2, 3]]) // Short
```

```
Out[8]/Short=
```

$$S\left[\text{Ar}[0, 1] \rightarrow \text{Ar}[0, 1] + Y\left[0, 1, 3, \text{AH}\left[-\frac{e^{\langle 1 \rangle} \langle 1 \rangle}{x[1]}\right]\right], \langle 2 \rangle, \text{Ar}[3, 0] \rightarrow \langle 1 \rangle\right]$$

```
In[9]:= (rhs1 = S[Exp[Ar[1, 3] + Ar[2, 3] + Y[1, 2, 3, bc]]) // Short
```

```
Out[9]/Short=
```

$$S\left[\text{Ar}[0, 1] \rightarrow \text{Ar}[0, 1] + Y\left[0, 1, 3, \text{AH}\left[\frac{\langle 1 \rangle}{x[1] + \langle 1 \rangle}\right]\right], \langle 1 \rangle, \langle 1 \rangle, \text{Ar}[3, 0] \rightarrow \langle 1 \rangle\right]$$

```
In[10]:= bch1 = bc /. First[HSolve[
  Coefficient[
    Ar[0, 1] // lhs1,
    Y[0, 1, 3]
  ] == Coefficient[
    Ar[0, 1] // rhs1,
    Y[0, 1, 3]
  ],
  bc
]]
```

$$\text{Out[10]= } \frac{-e^{x[1]} x[1] + e^{x[1]+x[2]} x[1] + x[2] - e^{x[1]} x[2]}{(-1 + e^{x[1]+x[2]}) x[1] x[2]}$$

```
In[11]:= Test[CanonicalForm[
  S[Exp[Ar[1, 3] + Ar[2, 3] + Y[1, 2, 3, AH[bch1]]]] == S[sigma[1, 3], sigma[2, 3]]
]]
```

Out[11]= True

#### ■ Compare with Kurlin

```
In[12]:= Test[Simplify[(bch1 /. {x[1] → x, x[2] → y}) == 1/y (1 - e^x - 1/x (x+y) / (e^{x+y} - 1))]]
```

Out[12]= True

#### ■ exp(x)exp(y)=exp(x+y)exp(bch2)

```
In[13]:= (lhs2 = S[sigma[1, 3], sigma[2, 3]]) // Short
```

Out[13]/Short=

$$S\left[\text{Ar}[0, 1] \rightarrow \text{Ar}[0, 1] + Y\left[0, 1, 3, \text{AH}\left[-\frac{e^{\langle 1 \rangle} \langle 1 \rangle}{x[1]}\right]\right], \langle 2 \rangle, \text{Ar}[3, 0] \rightarrow \langle 1 \rangle\right]$$

In[14]= rhs2 = S[Exp[Ar[1, 3] + Ar[2, 3]]] \*\* S[Exp[Y[1, 2, 3, bc2]]]

Out[14]= 
$$S \left[ \text{Ar}[0, 1] \rightarrow \text{Ar}[0, 1] + Y \left[ 0, 1, 3, \text{AH} \left[ - \frac{e^{-x[1]-x[2]} \left( 1 - e^{x[1]+x[2]} + bc2 e^{x[1]+x[2]} x[1] x[2] + bc2 e^{x[1]+x[2]} x[2]^2 \right)}{x[1] + x[2]} \right] \right], \right.$$

$$\text{Ar}[0, 2] \rightarrow \text{Ar}[0, 2] + Y \left[ 0, 2, 3, \right.$$

$$\text{AH} \left[ - \frac{e^{-x[1]-x[2]} \left( -1 + e^{x[1]+x[2]} + bc2 e^{x[1]+x[2]} x[1]^2 + bc2 e^{x[1]+x[2]} x[1] x[2] \right)}{x[1] + x[2]} \right] \right], \text{Ar}[0, 3] \rightarrow$$

$$\text{Ar}[0, 3] + Y \left[ 0, 1, 3, \text{AH} \left[ - \frac{e^{-x[1]-x[2]} \left( 1 - e^{x[1]+x[2]} + bc2 e^{x[1]+x[2]} x[1] x[2] + bc2 e^{x[1]+x[2]} x[2]^2 \right)}{x[1] + x[2]} \right] \right] \right] +$$

$$Y \left[ 0, 2, 3, \text{AH} \left[ \frac{e^{-x[1]-x[2]} \left( -1 + e^{x[1]+x[2]} + bc2 e^{x[1]+x[2]} x[1]^2 + bc2 e^{x[1]+x[2]} x[1] x[2] \right)}{x[1] + x[2]} \right] \right],$$

$$\text{Ar}[3, 0] \rightarrow \text{Ar}[3, 0] + Y \left[ 1, 2, 0, \text{AH} \left[ - \left( \left( x[1] - e^{x[2]} x[1] - e^{x[2]} x[2] + e^{x[1]+x[2]} x[2] + \right. \right. \right.$$

$$\left. \left. bc2 e^{x[1]+x[2]} x[1]^2 x[2] + bc2 e^{x[1]+x[2]} x[1] x[2]^2 \right) x[3] \right] / \left( x[1] x[2] \left( x[1] + x[2] \right) \right) \right] \right] +$$

$$Y \left[ 1, 3, 0, \text{AH} \left[ \frac{e^{x[2]} \left( -1 + e^{x[1]} \right)}{x[1]} \right] \right] + Y \left[ 2, 3, 0, \text{AH} \left[ \frac{-1 + e^{x[2]}}{x[2]} \right] \right] \right]$$

In[15]= bch2 = bc2 /. First[HSolve[  
Coefficient[  
Ar[0, 1] // lhs2,  
Y[0, 1, 3]  
] == Coefficient[  
Ar[0, 1] // rhs2,  
Y[0, 1, 3]  
],  
bc2  
]]

Out[15]= 
$$\frac{e^{-x[1]-x[2]} \left( -e^{x[1]} x[1] + e^{x[1]+x[2]} x[1] + x[2] - e^{x[1]} x[2] \right)}{x[1] x[2] \left( x[1] + x[2] \right)}$$

In[16]= Simplify[bch2 /. {x[1] -> x, x[2] -> y}]

Out[16]= 
$$\frac{e^{-x-y} \left( e^{x+y} x + y - e^x \left( x + y \right) \right)}{x y \left( x + y \right)}$$

■ exp(x)exp(y)exp(-x)exp(-y)=exp(bch3)

In[17]= (lhs3 = S[sigma[1, 3], sigma[2, 3], sigbar[1, 3], sigbar[2, 3]]) // Short

Out[17]//Short=

$$S \left[ \text{Ar}[0, 1] \rightarrow \text{Ar}[0, 1] + Y \left[ 0, 1, 3, \text{AH} \left[ \frac{\langle\langle 1 \rangle\rangle \langle\langle 1 \rangle\rangle}{x[1]} \right] \right], \langle\langle 2 \rangle\rangle, \text{Ar}[3, 0] \rightarrow \langle\langle 1 \rangle\rangle \right]$$

In[18]= rhs3 = S[Exp[Y[1, 2, 3, bc3]]]

Out[18]= 
$$S \left[ \text{Ar}[0, 1] \rightarrow \text{Ar}[0, 1] + Y \left[ 0, 1, 3, \text{AH} \left[ bc3 x[2] \right] \right], \text{Ar}[0, 2] \rightarrow \text{Ar}[0, 2] + Y \left[ 0, 2, 3, \text{AH} \left[ -bc3 x[1] \right] \right], \right.$$

$$\text{Ar}[0, 3] \rightarrow \text{Ar}[0, 3] + Y \left[ 0, 1, 3, \text{AH} \left[ -bc3 x[2] \right] \right] + Y \left[ 0, 2, 3, \text{AH} \left[ bc3 x[1] \right] \right],$$

$$\left. \text{Ar}[3, 0] \rightarrow \text{Ar}[3, 0] + Y \left[ 1, 2, 0, \text{AH} \left[ -bc3 x[3] \right] \right] \right]$$

```
In[19]:= bch3 = bc3 /. First[HSolve[
  Coefficient[
    Ar[0, 1] // lhs3,
    Y[0, 1, 3]
  ] == Coefficient[
    Ar[0, 1] // rhs3,
    Y[0, 1, 3]
  ],
  bc3
]]
```

```
Out[19]= 
$$\frac{(-1 + e^{x[1]}) (-1 + e^{x[2]})}{x[1] x[2]}$$

```

```
In[20]:= Simplify[bch3 /. {x[1] → x, x[2] → y}]
```

```
Out[20]= 
$$\frac{(-1 + e^x) (-1 + e^y)}{x y}$$

```

```
In[21]:= Test[CanonicalForm[
  S[Exp[Y[1, 2, 3, AH[bch3]]]] == S[sigma[1, 3], sigma[2, 3], sigbar[1, 3], sigbar[2, 3]]
]]
```

```
Out[21]= True
```